Research Memorandum 1

DISPERSION-LIMITED ERROR PROBABILITIES FOR CORRELATED FSK DIVERSITY RECEPTIONS

By: ROBERT F. DALY

Prepared for:

DEFENSE COMMUNICATIONS AGENCY
WASHINGTON, D.C. 20305

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ABSTRACT

The irreducible binary error probability of an FSK system *with memory* operating over a dispersive channel is analyzed. Binary error probability is obtained as a function of both time-delay and Doppler spread for a simple HF channel model. In addition, the effectiveness of *correlated* diversity receptions in combatting channel dispersion is investigated.

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I INTRODUCTION

This memorandum investigates the irreducible error probability of an FSK communication system that does not employ energy quenching in the detection process. In contrast to the incoherent matched-filter model (an energy-quenching device) assumed in the past,[1,2] our model more closely approximates existing systems that, in effect, employ narrowband filters with infinite memory. Thus we examine system performance in the presence of channel dispersion (at infinite signal-to-noise ratio) when, at any given time, the energy stored in the detection filters is a function of the entire history of both the channel variations and the transmitted signal. In addition, the effectiveness of *correlated* dual-diversity reception in combatting channel dispersion is investigated.

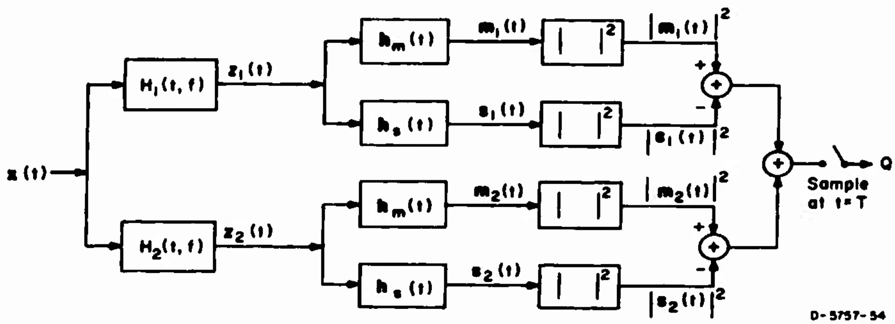

FIG. 1 DUAL-DIVERSITY FSK COMMUNICATION SYSTEM

Figure 1 depicts the communication system under consideration. Boldface notation indicates the complex envelope of the corresponding real narrowband signal. For example, the real signal

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1. R. F. Daly, "Analysis of Multipath Effects on FSK Error Probability for a Simple HF Channel Model," Research Memorandum 1, Contract SD-189, SRI Project 4554, Stanford Research Institute, Menlo Park, California (February 1964).

2. P. A. Bello and B. D. Nelin, "The Influence of Fading Spectrum on the Binary Error Probability of Incoherent and Differentially Coherent Matched Filter Receivers," *IRE Trans.*, PGCS-10, pp. 160-168 (June 1962).

$$x(t) = \mathrm{Re}\ \{\chi(t)e^{i2\pi f_0 t}\}$$

is the on-the-air transmission of an FSK system, and

$$z_1(t) = \mathrm{Re}\ \{\mathbf{z}_1(t)e^{i2\pi f_0 t}\}$$

is the real signal received in one of the branches of the dual-diversity receiving system. The diversity channels are modeled as two randomly time-varying linear filters with, in general, correlated Gaussian random-field transfer functions, $H_1(t,f)$ and $H_2(t,f)$.[3] If $\mathbf{X}(f)$ is the Fourier transform of $\chi(t)$, then

$$\mathbf{z}_i(t) = \int H_i(t,f)\mathbf{X}(f)e^{i2\pi f t}df, \qquad i = 1,2 \quad . \tag{1}$$

The functions $\mathbf{h}_m(t)$ and $\mathbf{h}_s(t)$ are the complex envelopes of the impulse responses of the narrowband filters centered at the mark and space frequencies, $f_0 - \Delta$ and $f_0 + \Delta$, respectively,

$$\mathbf{h}_m(t) = \begin{cases} Be^{-\alpha t}e^{i2\pi\Delta(T-t)} & , \quad t \geq 0 \\ 0 & \quad t < 0 \end{cases} \tag{2a}$$

$$\mathbf{h}_s(t) = \begin{cases} Be^{-\alpha t}e^{-i2\pi\Delta(T-t)} & , \quad t \geq 0 \\ 0 & , \quad t < 0 \end{cases} \quad . \tag{2b}$$

After narrowband filtering at the mark and space frequencies, each diversity branch forms the difference of the mark and space square-law detected envelopes. The diversity outputs are then summed and sampled at time T, yielding the decision variable Q.

In one of the cases to be investigated, we assume $\mathbf{X}(t)$ to have the following form:

$$\mathbf{X}(t) = \begin{cases} Ae^{i2\pi\Delta t} & \quad t \leq 0 \\ Ae^{-i2\pi\Delta t} & , \quad 0 \leq t \leq T \\ Ae^{i2\pi\Delta(t-2T)} & , \quad T \leq t \end{cases} \quad . \tag{3}$$

3. R. F. Daly, "On Modeling the Time-Varying Frequency-Selective Radio Channel," Technical Report 2—Part II, Contract DA 36-039 SC-90859, SRI Project 4172, Stanford Research Institute, Menlo Park, California (July 1964).

We obtain this FSK transmission when we assume mark transmitted in the interval (O,T) and space transmitted for all t outside this interval. At time $t = T$, the receiver obtains the statistic

$$Q = |\mathbf{m}_1(T)|^2 - |\mathbf{s}_1(T)|^2 + |\mathbf{m}_2(T)|^2 - |\mathbf{s}_2(T)|^2 \tag{4}$$

and declares mark transmitted in the interval (O,T) if $Q \geq 0$. Thus, assuming mark and space to be equiprobable messages, the probability of error P_e, for $\mathbf{X}(t)$ transmitted is the probability that Q is less than zero,

$$P_e = P[Q < 0] \quad . \tag{5}$$

II CHARACTERISTIC FUNCTION OF THE DECISION STATISTIC

Decisions are based on the statistic

$$Q = |m_1|^2 - |s_1|^2 + |m_2|^2 - |s_2|^2 \quad , \tag{6}$$

where we have suppressed the argument T appearing in Eq. (4). The statistic Q is a quadratic form in complex Gaussian random variables. Employing a result due to Turin,[4] we obtain an expression for the characteristic function of Q.

Let V be the vector of mark and space filter outputs,

$$V^T = [m_1 s_1 m_2 s_2] \quad , \tag{7}$$

and G the Hermitian matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad , \tag{8}$$

then

$$Q = V^{T^*} G V \tag{9}$$

is a Hermitian quadratic form in the complex Gaussian variables, m_1, s_1, m_2, s_2. Turin showed that Q has a characteristic function of the form

$$\Phi_Q(t) = \prod_n (1 - it\lambda_n)^{-1} \quad . \tag{10}$$

The λ_n are the eigenvalues of a matrix defined as follows: Let K_V be the covariance matrix of the vector V,

$$K_V = E[VV^{T^*}] \quad , \tag{11}$$

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4. G. L. Turin, "The Characteristic Function of Hermitian Quadratic Forms in Complex Normal Variables," *Biometrika*, Vol. 47, pp. 199-201 (June 1960).

then the λ_n in Eq. (10) are the eigenvalues of the matrix

$$K_V G = E[VV^{T*}G] = E\begin{bmatrix} m_1m_1^* & -m_1s_1^* & m_1m_2^* & -m_1s_2^* \\ s_1m_1^* & -s_1s_1^* & s_1m_2^* & -s_1s_2^* \\ m_2m_1^* & -m_2s_1^* & m_2m_2^* & -m_2s_2^* \\ s_2m_1^* & -s_2s_1^* & s_2m_2^* & -s_2s_2^* \end{bmatrix} . \qquad (12)$$

In this memorandum we evaluate the eigenvalues of the matrix $K_V G$, invert the characteristic function $\Phi_Q(t)$, and obtain an expression for the probability of error P_e.

III DIVERSITY CHANNEL MODELS

Two correlated Gaussian random-field transfer functions, $H_1(t,f)$ and $H_2(t,f)$, serve to model the diversity channels. We assume that the random fields are homogeneous and identically distributed,

$$E[H_1^*(t,f)H_1(t+\alpha,f+\beta)] = E[H_2^*(t,f)H_2(t+\alpha,f+\beta)] = R_H(\alpha,\beta) \quad , \tag{13a}$$

and that the cross correlation between the channels is simply given by

$$E[H_1^*(t,f)H_2(t+\alpha,f+\beta)] = \rho R_H(\alpha,\beta) \quad , \tag{13b}$$

where

$$0 \le |\rho| \le 1 \quad .$$

Observe that Eq. (13b) can be interpreted in terms of a three-dimensional random field (for example, we may consider diversity-antenna separation, d, as the third dimension) in which the third-dimensional dependence can be factored from the three-dimensional autocorrelation function $\mathcal{R}_H(d,\alpha,\beta)$:

$$\mathcal{R}_H(d,\alpha,\beta) = C(d)R_H(\alpha,\beta) = \rho R_H(\alpha,\beta) \quad . \tag{14}$$

The diversity correlation coefficient ρ as formulated in Eq. (13b) is defined in terms of complex envelopes and can itself be complex. In practice, one usually deals with the correlation between real envelopes. In what follows we relate the magnitude of ρ to an easily measurable quantity. the correlation coefficient between the squared envelopes at zero time lag and zero frequency shift:

$$\gamma = \frac{\mathrm{Cov}[|H_1|^2, |H_2|^2]}{\mathrm{Var}^{1/2}[|H_1|^2]\mathrm{Var}^{1/2}[|H_2|^2]} \tag{15}$$

where the t and f dependence has been suppressed in the $H_i(t,f)$.

$$\mathrm{Cov}[|H_1|^2, |H_2|^2] = E[|H_1|^2|H_2|^2] - E[|H_1|^2]E[|H_2|^2]$$

$$\begin{aligned} E[|H_1|^2|H_2|^2] &= E[H_1^*H_1H_2H_2^*] \\ &= E[H_1^*H_1]E[H_2H_2^*] + E[H_1^*H_2^*]E[H_1H_2] + E[H_1^*H_2]E[H_1H_2^*] \\ &= R_H^2(0,0) + 0 + |\rho|^2R_H^2(0,0) \\ &= (1 + |\rho|^2)R_H^2(0,0) \quad . \end{aligned}$$

$$\mathrm{Cov}[|H_1|^2, |H_2|^2] = (1 + |\rho|^2)R_H^2(0,0) - R_H^2(0,0) = |\rho|^2R_H^2(0,0) \quad .$$

$$\mathrm{Var}[|H_2|^2] = \mathrm{Var}[|H_1|^2] = E[|H_1|^4] - E[|H_1|^2] \quad .$$

$$\begin{aligned} E[|H_1|^4] &= E[H_1H_1^*H_1H_1^*] \\ &= E[H_1H_1^*]E[H_1H_1^*] + E[H_1H_1]E[H_1^*H_1^*] + E[H_1H_1^*]E[H_1H_1^*] \\ &= 2R_H^2(0,0) \quad . \end{aligned}$$

$$\mathrm{Var}[|H_1|^2] = 2R_H^2(0,0) - R_H^2(0,0) = R_H^2(0,0) \quad .$$

From Eq. (15), we obtain

$$\gamma = \frac{|\rho|^2R_H^2(0,0)}{R_H^2(0,0)} = |\rho|^2 \quad . \tag{16}$$

To simplify the evaluation and interpretation of the FSK error-probability performance, we restrict our attention to the case in which ρ is a real quantity. In this case, the error probability will depend on ρ^2, the squared-envelope, diversity correlation coefficient.

IV FILTER OUTPUT COVARIANCES

The elements of the matrix $K_V G$ (omitting sign) are the elements of the matrix K_V, the covariance matrix associated with the filter outputs. In lieu of the prior notation, $\mathfrak{m}_1$, $\mathfrak{s}_1$, $\mathfrak{m}_2$, $\mathfrak{s}_2$, it is convenient at this point to express the four filter outputs as follows,

$$V_i(r) = \int_{-\infty}^{T} \mathbf{z}_i(t)\mathfrak{h}_r(T - t)dt \quad \begin{cases} i = 1,2 \\ r = m,s \end{cases} , \tag{17}$$

where the index i indicates the diversity branch and r the mark or space channel. All terms of the covariance matrix can be written in the form

$$E[V_i^*(r_1)V_j(r_2)] = \int_{-\infty}^{T}\int_{-\infty}^{T} E[\mathbf{z}_i^*(t_1)\mathbf{z}_j(t_2)]\mathfrak{h}_{r_1}^*(T - t_1)\mathfrak{h}_{r_2}(T - t_2)dt_1dt_2 \quad . \tag{18}$$

Equations (1) and (13) imply that

$$E[\mathbf{z}_i^*(t_1)\mathbf{z}_j(t_2)] = k_{ij}\iint \mathbf{x}^*(t_1 - \tau)\mathbf{x}(t_2 - \tau)e^{i2\pi\lambda(t_2-t_1)}S_v(\lambda,\tau)d\lambda d\tau , \tag{19}$$

where

$$k_{ij} = \begin{cases} 1 & \text{if } i = j \\ \rho & \text{if } i \neq j \end{cases}$$

and

$$S_v(\lambda,\tau) = \iint e^{-i2\pi(\lambda\alpha-\tau\beta)}R_H(\alpha,\beta)d\alpha d\beta$$

is the channel scattering function.

The symmetric two-path case

$$S_\nu(\lambda,\tau) = \frac{R_H(0,0)}{2}\left\{\delta(\lambda-\lambda')\delta(\tau-\tau') + \delta(\lambda+\lambda')\delta(\tau+\tau')\right\}, \qquad (20)$$

is particularly useful in analyzing systems operating over HF channels. The channel is modeled as two discrete paths spaced $2\tau'$ in time delay and $2\lambda'$ in Doppler shift. Let

$$A_{xr}(\lambda,\tau) = \int_{-\infty}^{T} \mathbf{x}(t-\tau)\mathbf{h}_r(T-t)e^{i2\pi\lambda t}dt\,, \qquad r = m,s, \qquad (21)$$

then, for the symmetric two-path case,

$$E[V_i^*(r_1)V_j(r_2)] = k_{ij}\frac{R_H(0,0)}{2}\left\{A_{xr_1}^*(\lambda',\tau')A_{xr_2}(\lambda',\tau') + A_{xr_1}^*(-\lambda',-\tau')A_{xr_2}(-\lambda',-\tau')\right\}. \qquad (22)$$

V THE EIGENVALUES OF $K_\nu G$

Let

$$c_{r_1 r_2} = E[V_i^*(r_1)V_i(r_2)] \tag{23a}$$

for $r_1 = m,s$; $r_2 = m,s$ and $i = 1,2$, and

$$C = \begin{bmatrix} c_{mm} & -c_{ms} \\ c_{sm} & -c_{ss} \end{bmatrix} \quad ; \tag{23b}$$

then it follows from Eq. (22) that the matrix $K_\nu G$ can be written

$$K_\nu G = \begin{bmatrix} C & \rho C \\ \rho C & C \end{bmatrix} \quad . \tag{24}$$

We seek solutions (λ and Y) to the equation

$$K_\nu G Y = \lambda Y \quad . \tag{25}$$

By setting

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

and using Eq. (24), Eq. (25) becomes

$$\begin{bmatrix} C & \rho C \\ \rho C & C \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \lambda \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad . \tag{26}$$

Let the matrix C have eigenvectors X_i and eigenvalues ω_i, $i = 1,2$, then

$$CX_i = \omega_i X_i \quad , \quad i = 1,2 \quad . \tag{27}$$

In Eq. (26) let $Y_1 = Y_2 = X_\iota$,

$$\begin{bmatrix} (1 + \rho)CX_\iota \\ \\ (1 + \rho)CX_\iota \end{bmatrix} = \lambda \begin{bmatrix} X_\iota \\ \\ X_\iota \end{bmatrix}$$

$$\begin{bmatrix} (1 + \rho)\omega_\iota X_\iota \\ \\ (1 + \rho)\omega_\iota X_\iota \end{bmatrix} = \lambda \begin{bmatrix} X_\iota \\ \\ X_\iota \end{bmatrix} \quad ,$$

thus, if C has eigenvalues ω_1 and ω_2, then $K_V G$ has eigenvalues $(1 + \rho)\omega_1$ and $(1 + \rho)\omega_2$. In Eq. (26) let $Y_1 = -Y_2 = X_\iota$

$$\begin{bmatrix} (1 - \rho)CX_\iota \\ \\ -(1 - \rho)CX_\iota \end{bmatrix} = \lambda \begin{bmatrix} X_\iota \\ \\ -X_\iota \end{bmatrix}$$

$$\begin{bmatrix} (1 - \rho)\omega_\iota X_\iota \\ \\ -(1 - \rho)\omega_\iota X_\iota \end{bmatrix} = \lambda \begin{bmatrix} X_\iota \\ \\ -X_\iota \end{bmatrix}$$

thus, if C has eigenvalues ω_1 and ω_2, then $K_V G$ has eigenvalues $(1 - \rho)\omega_1$ and $(1 - \rho)\omega_2$. Let the four eigenvalues of $K_V G$ be denoted by λ_ι, $\iota = 1,2,3,4$ and the two eigenvalues of C by ω_ι, $\iota = 1,2$, then:

$$\begin{aligned} \lambda_1 &= (1 + \rho)\omega_1 \\ \lambda_2 &= (1 - \rho)\omega_1 \\ \lambda_3 &= (1 + \rho)\omega_2 \\ \lambda_4 &= (1 - \rho)\omega_2 \quad . \end{aligned} \tag{28}$$

To evaluate ω_1 and ω_2, we find the roots of the quadratic equation,

$$|C - \omega I| = 0$$

$$\omega^2 - (c_{mm} - c_{ss})\omega + |c_{ms}|^2 - c_{mm}c_{ss} = 0 \quad , \tag{29}$$

$$\begin{matrix}\omega_1 \\ \omega_2\end{matrix} = \frac{(c_{mm} - c_{ss})}{2} \pm \frac{1}{2} \left\{(c_{mm} + c_{ss})^2 - 4|c_{ms}|^2\right\}^{1/2} \quad . \tag{30}$$

One can verify that the above roots are real and that ω_1 is positive and ω_2 negative. The eigenvalues λ_1 and λ_2 are real and positive, and the eigenvalues λ_3 and λ_4 are real and negative.

VI ERROR PROBABILITY EVALUATION

The probability density of the decision statistic Q is found by inverting the characteristic function,

$$f_Q(q) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-itq}\phi_Q(t)\,dt$$

$$f_Q(q) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} e^{-sq}\phi_Q(-is)\,ds$$

$$f_Q(q) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} e^{-sq}\prod_{n=1}^{4}(1-\lambda_n s)^{-1}\,ds \quad . \tag{31}$$

For $q < 0$, the integrand in Eq. (31) goes to zero along an infinite semi-circle in the left-hand plane; for $q > 0$, the integrand goes to zero along an infinite semicircle in the right-half plane. Thus, letting r_n denote the residue of the integrand at the pole $s = 1/\lambda_n$ and evaluating the integral in Eq. (31) by residues, we obtain:

$$f_Q(q) = \begin{cases} r_1 + r_2 \ , & \text{for} \quad q > 0 \\ r_3 + r_4 \ , & \text{for} \quad q < 0 \end{cases} \ , \tag{32}$$

where

$$r_k = \lambda_k^2 e^{-q/\lambda_k}\prod_{n\neq k}(\lambda_n - \lambda_k)^{-1} \quad . \tag{33}$$

(The eigenvalues λ_1 and λ_2 are positive, and the eigenvalues λ_3 and λ_4 are negative.) Thus, given that mark is transmitted in the interval (O,T), the probability of error can be written

$$P_e = P[Q < 0] = \int_{-\infty}^{0} f_Q(q)\,dq \quad ,$$

$$P_e = -\lambda_3^3 \prod_{n \neq 3} (\lambda_n - \lambda_3)^{-1} - \lambda_4^3 \prod_{n \neq 4} (\lambda_n - \lambda_4)^{-1} \quad . \tag{34}$$

The expression for the probability of error can be put in terms of the elements of the matrix C and the parameter ρ. Let

$$X = \omega_1 - \omega_2 = \{(c_{mm} + c_{ss})^2 - 4|c_{ms}|^2\}^{1/2} \tag{35a}$$

$$Y = \omega_1 + \omega_2 = c_{mm} - c_{ss} \quad , \tag{35b}$$

then

$$P_e = \frac{(Y - X)^2[(1 + \rho^2)Y + 2X]}{4X[X^2 - \rho^2 Y^2]} \quad . \tag{36}$$

VII THE FILTER-SIGNAL CROSS-AMBIGUITY FUNCTIONS

The functions,

$$A_{xm}(\lambda,\tau) = \int_{-\infty}^{T} \mathrm{x}(t-\tau)\mathrm{h}_m(T-t)e^{i2\pi\lambda t}dt \tag{37a}$$

$$A_{xs}(\lambda,\tau) = \int_{-\infty}^{T} \mathrm{x}(t-\tau)\mathrm{h}_s(T-t)e^{i2\pi\lambda t}dt \quad , \tag{37b}$$

appearing in Eq. (22) represent the outputs of the mark and space filters at time T when the channel introduces a time delay τ and a Doppler shift λ. In this section we document these functions for two different transmissions and the narrowband filters assumed in Eq. (2). Since, for the filters assumed in Eq. (2),

$$A_{xs}(\lambda,\tau) = A_{xm}(\lambda - 2\Delta,\tau) \quad ,$$

it is sufficient to display $A_{xm}(\lambda,\tau)$.

Let

$$\mathrm{x}(t) = \begin{cases} Ae^{i2\pi\Delta t} & , \ t \le 0 \\ Ae^{-i2\pi\Delta t} & , \ 0 \le t \le T \ , \\ Ae^{i2\pi\Delta(t-2T)} & , \ T \le t \end{cases} \tag{38}$$

then

$$A_{xm}(\lambda,\tau) = \frac{ABe^{-\alpha T}}{\alpha + i2\pi(\lambda + 2\Delta)} \exp\{[\alpha + i2\pi(\lambda + \Delta)]\tau\}$$

$$+ \frac{AB\exp\{-\alpha T + i2\pi\Delta\tau\}}{\alpha + i2\pi\lambda}[\exp\{(\alpha + i2\pi\lambda)T\} - \exp\{(\alpha + i2\pi\lambda)\tau\}] \ ,$$

for

$$0 \le \tau \le T \quad , \tag{39a}$$

$$A_{xm}(\lambda,\tau) = \frac{ABe^{-\alpha T}}{\alpha + i2\pi(\lambda + 2\Delta)} \exp\{[\alpha + i2\pi(\lambda + \Delta)]\tau\}$$

$$+ \frac{AB \exp\{-\alpha T + i2\pi\Delta\tau\}}{\alpha + i2\pi\lambda} \exp\{(\alpha + i2\pi\lambda)\tau\}[\exp\{(\alpha + i2\pi\lambda)\tau\} - 1]$$

$$+ \frac{AB \exp\{i2\pi(\lambda T - \Delta\tau)\}}{\alpha + i2\pi(\lambda + 2\Delta)} [1 - \exp\{[\alpha + i2\pi(\lambda + 2\Delta)]\tau\} \quad ,$$

for
$$-T \leq \tau \leq 0 \quad . \tag{39b}$$

A more realistic transmission than that of Eq. (38) is obtained by not assuming the transmitter to be in the space state for an infinite past.

Let

$$x(t) = \begin{cases} 0 & , \quad t \leq -T \\ Ae^{i2\pi\Delta t} & , \quad T \leq t \leq 0 \\ Ae^{-i2\pi\Delta t} & , \quad 0 \leq t \leq T \\ Ae^{i2\pi\Delta(t-2T)} & , \quad T \leq t \leq 2T \\ 0 & , \quad 2T \leq t \end{cases} \quad , \tag{40}$$

then $A_{xm}(\lambda,\tau)$ is obtained by subtracting the term

$$L_{xm}(\lambda,\tau) = \frac{AB \exp\{\alpha(\tau - 2T) + i2\pi[\tau(\Delta + \lambda) - T(\lambda + 2\Delta)]\}}{\alpha + i2\pi(\lambda + 2\Delta)}$$

from Eqs. (39a) and (39b).

The error probability behavior is independent of any normalization of the function $A_{xm}(\lambda,\tau)$. It is also interesting to observe that $A_{xm}(\lambda,\tau)$ depends only on the following normalized parameters:

αT, a normalized filter time constant,

ΔT, a normalized frequency shift,

λT, a normalized Doppler shift,

τ/T, a normalized time delay.

VIII CONCLUSIONS

Figures 2 through 9 summarize the asymptotic or irreducible error-probability behavior of a dual-diversity FSK system with memory. All figures pertain to the case in which the mark and space frequencies are separated by the inverse of signaling-element duration ($2\Delta T = 1$). The symmetric two-path case, described in Eq. (20), is assumed. The differerence in time delay between the two paths, divided by T, is referred to as the time-delay spread $2\sigma_\tau$. Similarly, the difference in Doppler shift between the two paths, multiplied by T, is referred to as the Doppler spread $2\sigma_\lambda$. The transmission described by Eq. (38) is referred to as the *infinite space past* case and the transmission described by Eq. (40) is referred to as the *one element space past* case.

It is interesting to observe that the time constant, $\alpha T = 2$, yields a significant improvement in performance over that obtained for $\alpha T = 1$. Also of interest is the fact that diversity is effective in combatting channel dispersion for correlations as high as 0.8.

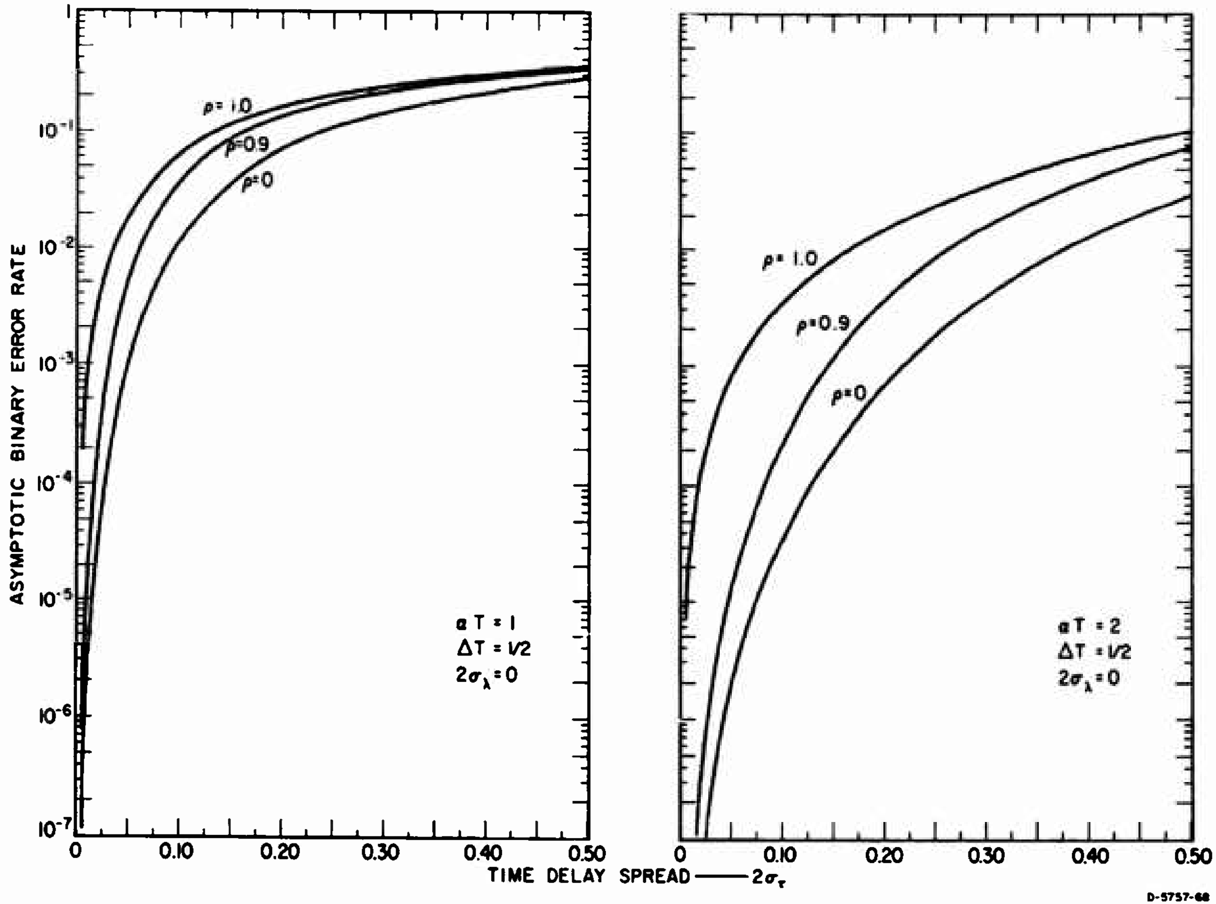

FIG. 2 ASYMPTOTIC BINARY ERROR RATE vs. $2\sigma_\tau$ INFINITE SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$

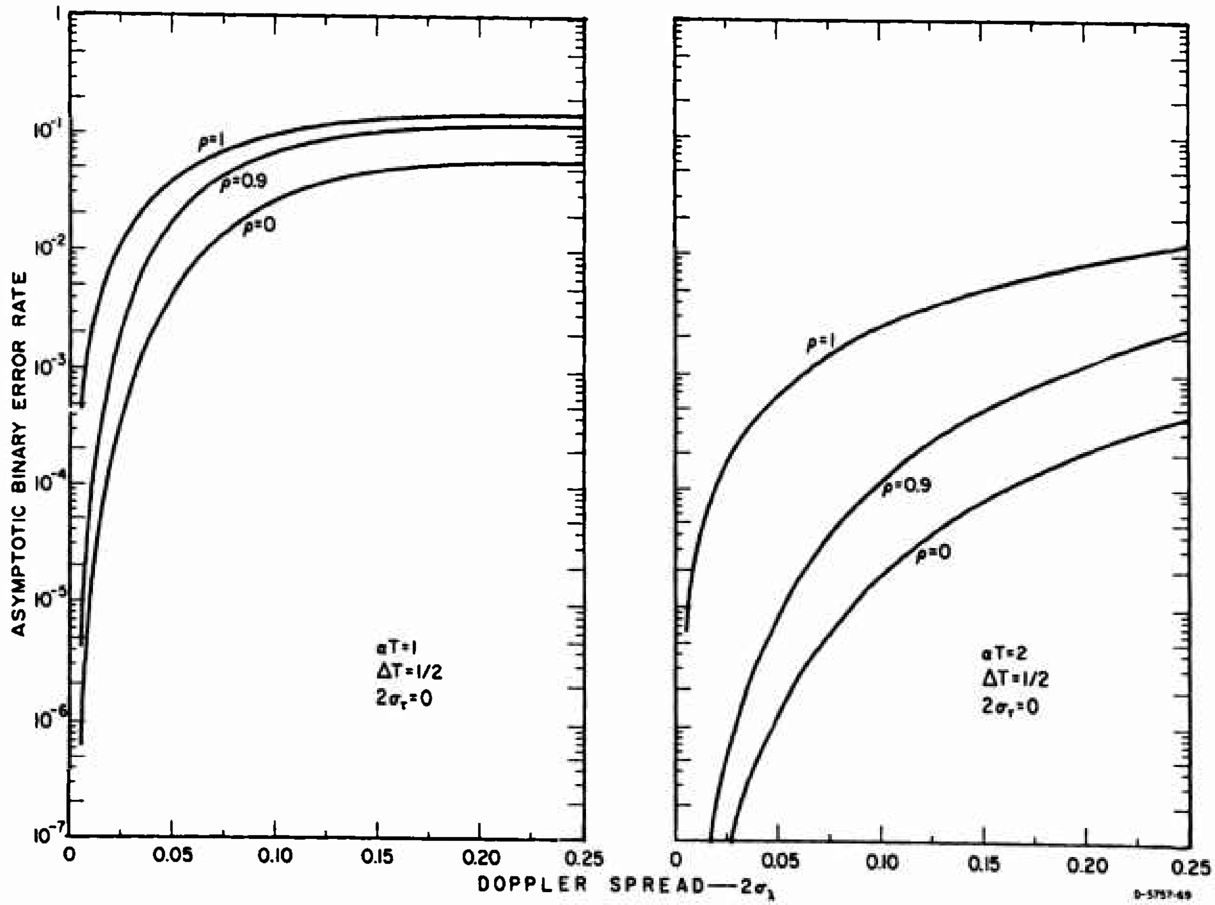

FIG. 3 ASYMPTOTIC BINARY ERROR RATE vs. $2\sigma_\lambda$ INFINITE SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$

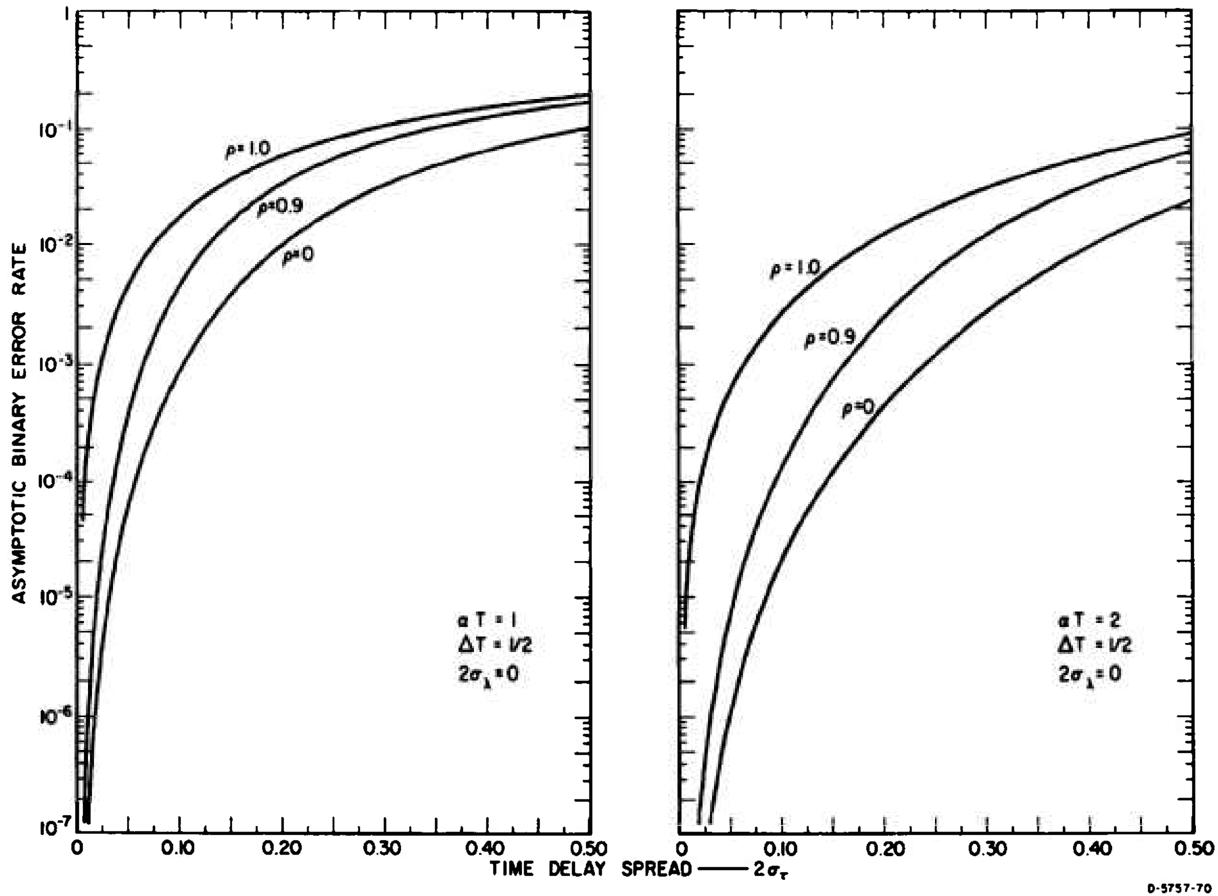

FIG. 4 ASYMPTOTIC BINARY ERROR RATE vs. $2\sigma_\tau$ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND αT 2

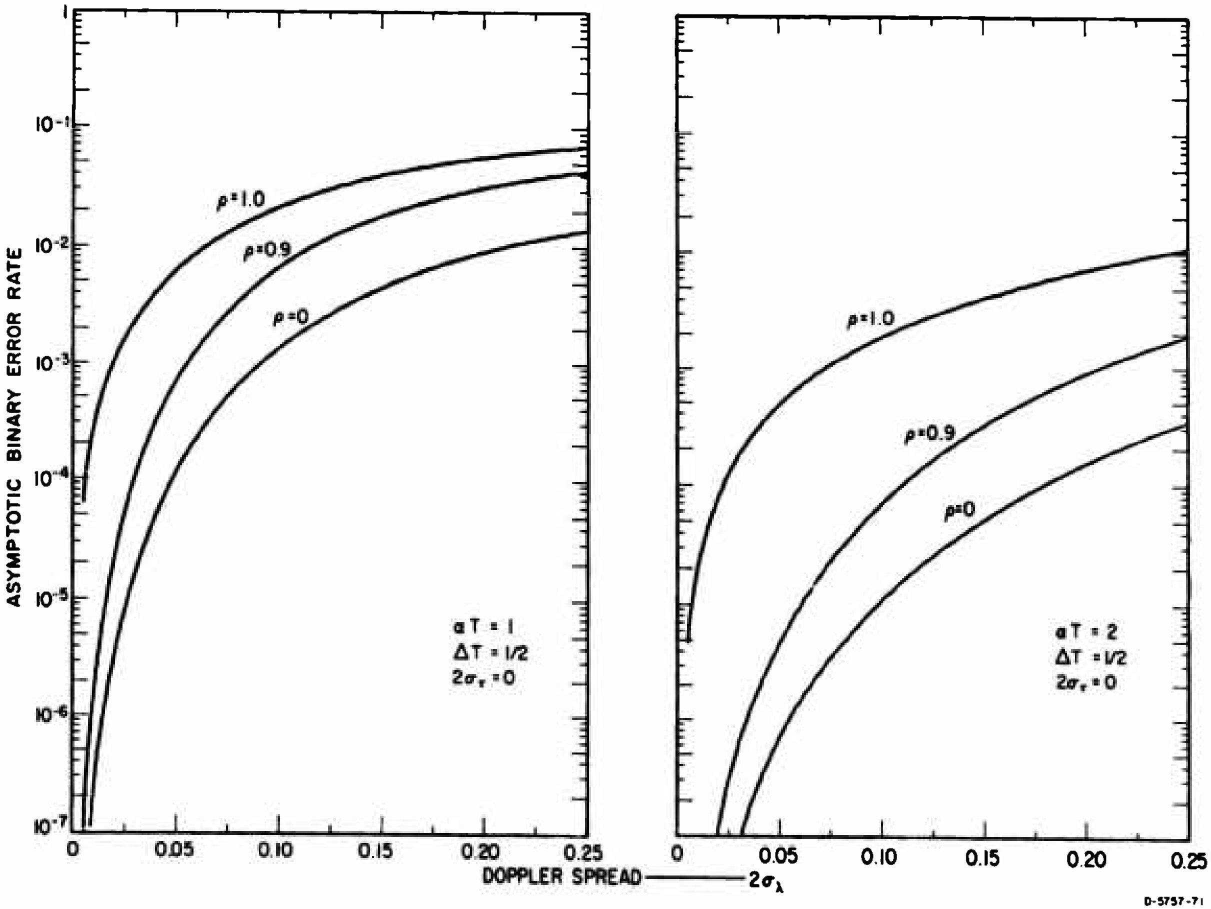

FIG. 5 ASYMPTOTIC BINARY ERROR RATE vs. $2\sigma_\lambda$ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$

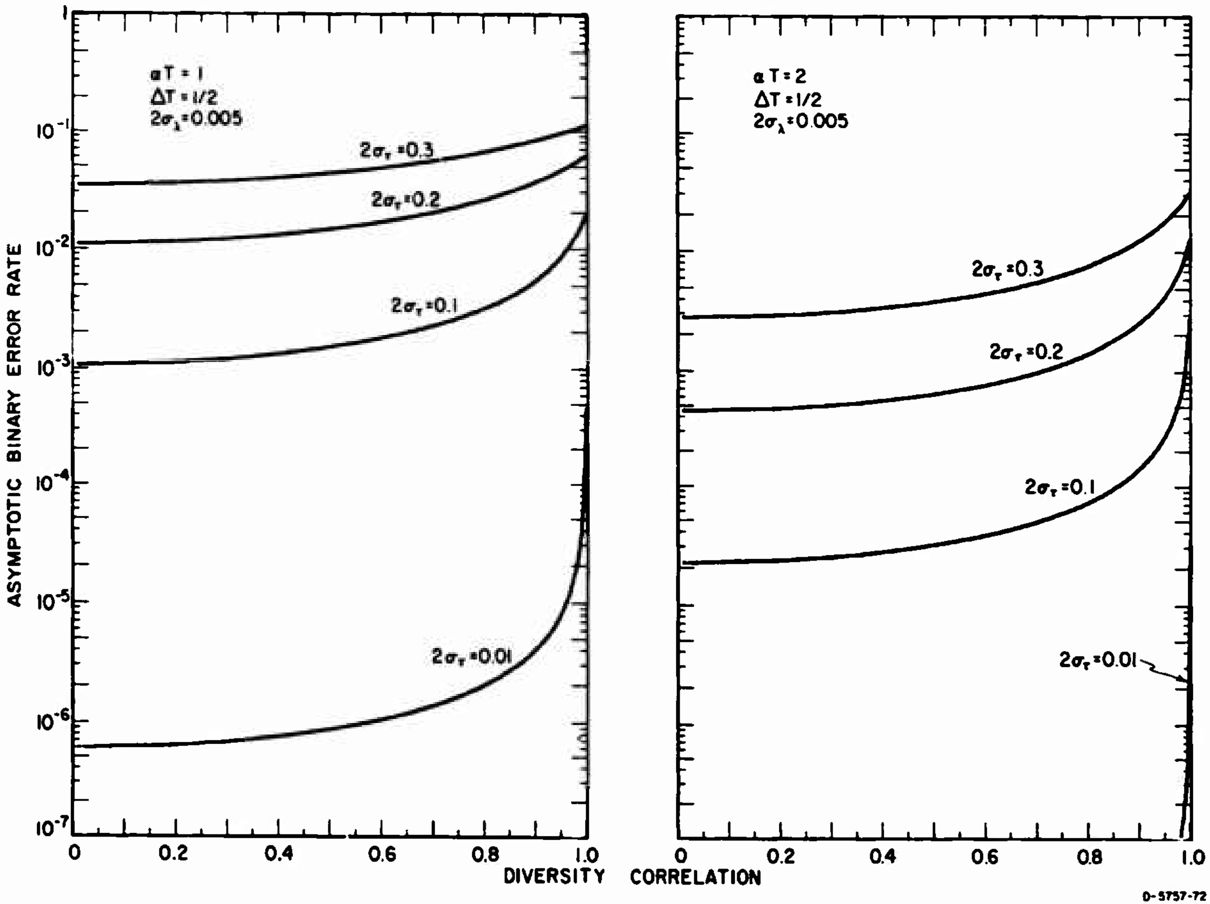

FIG. 6 ASYMPTOTIC BINARY ERROR RATE vs. ρ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$, $2\sigma_\lambda = 0.005$

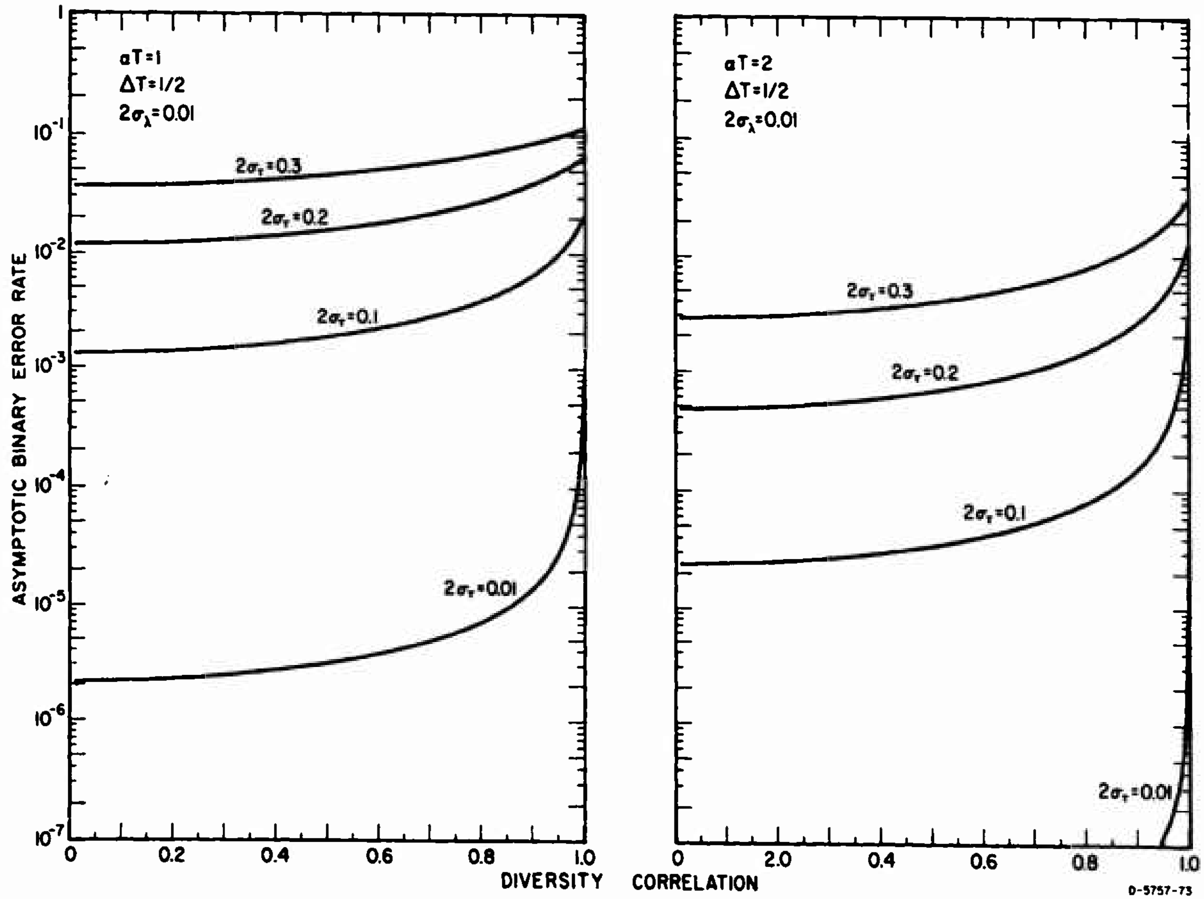

FIG. 7 ASYMPTOTIC BINARY ERROR RATE vs. ρ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$, $2\sigma_\lambda$ 0.01

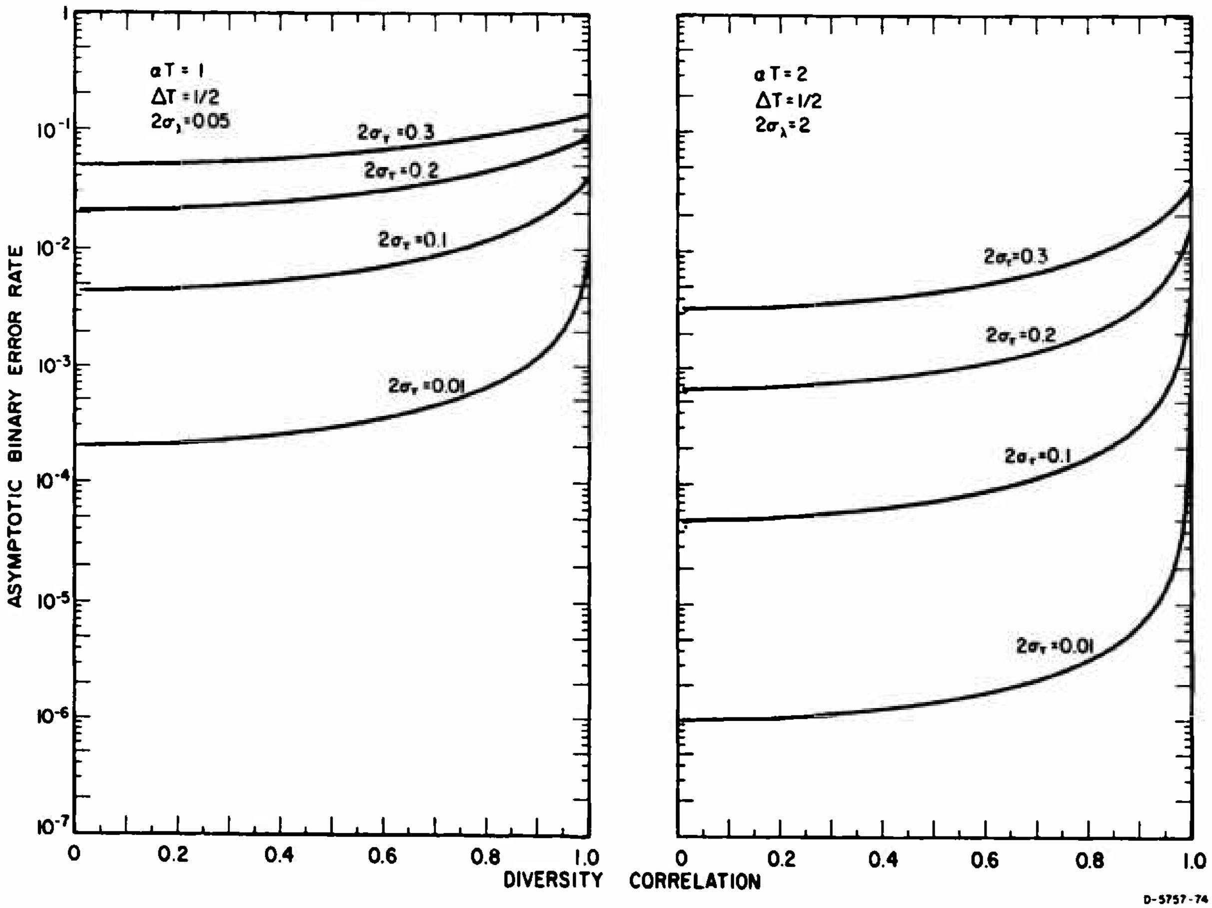

FIG. 8 ASYMPTOTIC BINARY ERROR RATE vs. ρ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$, $2\sigma_\lambda$ 0.05

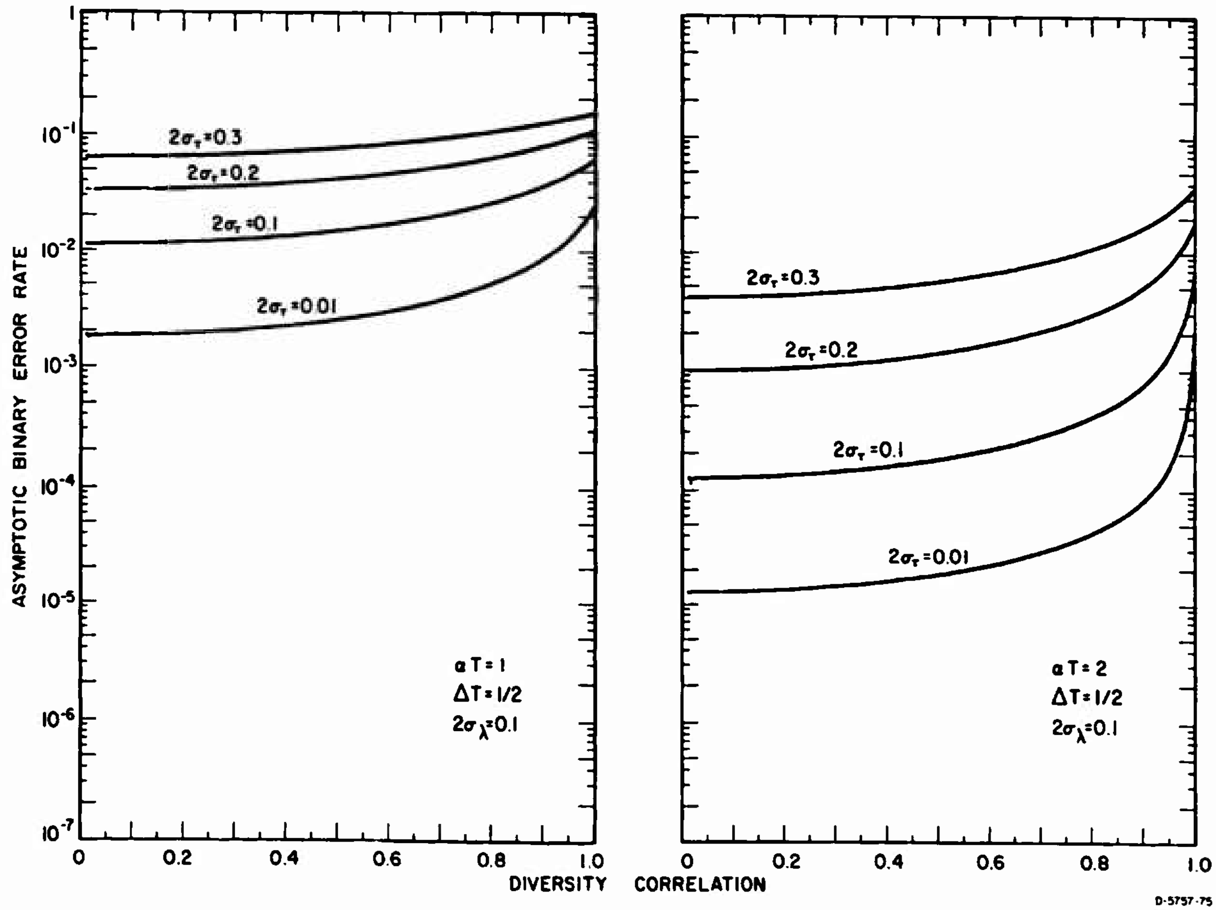

FIG. 9 ASYMPTOTIC BINARY ERROR RATE vs. ρ ONE ELEMENT SPACE PAST, $\alpha T = 1$ AND $\alpha T = 2$, $2\sigma_\lambda = 0.10$

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13. ABSTRACT

The irreducible binary error probability of an FSK system *with memory* operating over a dispersive channel is analyzed. Binary error probability is obtained as a function of both time-delay and Doppler spread for a simple HF channel model. In addition, the effectiveness of *correlated* diversity receptions in combatting channel dispersion is investigated.

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